OPTIMUM FEED COORDINATE IN A COLUMN SEPARATOR

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Analytic expressions are derived for the feed coordinate and flow division factor giving maximum separating power in a column-type apparatus.

In isotope separation, it is often necessary to devise separator cascades in column form, where the feed is input at an intermediate point [1]. It is complicated to optimize the column to improve the economic performance [1-3], and nonlinear programming is required [4]; it is thus best to develop methods for determining the optimum parameters approximetaly.

We consider separating a bindary isotope mixture having any concentration c_F for the component to be enriched in the feed flow F and determine what values of the coordinate ℓ for the feed and what value of the separation factor Θ will give maximum separating power δU.

The transport equations for the masses of target component in the depleted and enriched parts take the following form, where there is a transverse enrichment effect set up by an external field and which is multiplied by the length of the device on account of the mixture countercurrent, where in general the transport coefficients are dependent on the axial coordinate x [3]:

$$\frac{dc}{dz}=\frac{H^W(z)}{K^W(z)}c(1-c)+\frac{W}{K^W(z)}(c^- - c),\ 0\leqslant z\leqslant L-l,$$
$$\frac{dc}{dz}=\frac{H^P(z)}{K^P(z)}c(1-c)-\frac{P}{K^P(z)}(c^+ - c),\ L-l\leqslant z\leqslant L, \tag{1}$$

where the detailed forms of H(z) and K(z) are determined by the method and conditions.

We assume that the concentration limits for the target component or the concentration in the feed will be such that the c(1 − c) nonlinear term in (1) can be approximated as a linear combination a + bc [3, 5]. Then one solves (1) with the target-component mass-conservation equation

$$c_F=\Theta c^+ + (1-\Theta)c^-$$

and the boundary conditions

$$c=c^-\ \text{at}\ z=0,\ c=c^+\ \text{at}\ z=L,\ c=c_f\ \text{at}\ z=L-l$$

and assumes in the general case that c_F and the concentration c_f within the column at the feed point (z = L − ℓ) are not the same, which gives an expression for the total enrichment $\delta = c^+ - c^-$:

$$\frac{c^+ - c^-}{c_F(1-c_F)}=\tilde{\delta}, \tag{2}$$

in which

$$\tilde{\delta}=\frac{(Y-X)}{b[\Theta Y+(1-\Theta)X]},$$
$$c_f=\frac{XY\delta}{(Y-X)}-\frac{a}{b},$$
$$Y=1+b\exp\left[\int\limits_0^{L-l}p_1(z)\,dz\right]\int\limits_0^{L-l}H^W(z)\exp\left[-\int\limits_0^{z}p_1(z')\,dz'\right]\frac{dz}{K^W(z)},$$

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$$X = 1 - b \int_{L-l}^{L} H^P(z) \exp\left[-\int_{L-l}^{z} p_2(z')\,dz'\right] \frac{dz}{K^P(z)}, \quad (3)$$

$$p_1 = \frac{H^W(z)\,b - W}{K^W(z)},$$

$$p_2 = \frac{H^P(z)\,b + P}{K^P(z)}.$$

The separating power is [2]

$$\delta U = PV(c^+) + WV(c^-) - FV(c_F). \quad (4)$$

Eq. (4) can be expanded as a Taylor series near c_F:

$$\delta U = \frac{\Theta(1-\Theta)}{2} F\tilde{\delta}^2 I_0 I_1, \quad (5)$$

in which

$$I_0 = [c_F(1-c_F)]^2 \frac{d^2V}{dc_F^2},$$

$$I_1 = \sum_{n=2}^{\infty} \frac{2}{n!} \tilde{\delta}^{(n-2)} [(1-\Theta)^{(n-1)} + (-1)^n \Theta^{(n-1)}] \frac{d^{(n-2)}V}{dc_F^{(n-2)}} = \sum_{n=2}^{\infty} a_n.$$

We differentiate (5) with respect to $y = \ell/L$:

$$\frac{\partial \delta U}{\partial y} = \frac{\Theta(1-\Theta)}{2} F\tilde{\delta} \frac{\partial \tilde{\delta}}{\partial y} I_0 I_2, \quad (6)$$

where $I_2 = \sum_{n=2}^{\infty} n a_n$.

Equation (6) shows that series I_2 is sign-varying, so the separating power has a maximum with respect to y for

$$\frac{\partial \tilde{\delta}}{\partial y} = 0. \quad (7)$$

We average $H^F(z)$, $K^P(z)$ and $H^W(z)$, $K^W(z)$ over the heights of the tap-off and discharge parts of the column and take them as correspondingly $\overline{H}^P$, $\overline{K}^P$ and $\overline{H}^W$, $\overline{K}^W$. One substitutes for $\tilde{\delta}$ from (3) into (7) and performs transformations to get a transcendental equation for the optimum feed coordinate:

$$y_0 = \frac{1 - \Theta - b\frac{\overline{H}^W}{F} + \overline{K}^W \frac{\ln A}{FL}}{1 - \Theta\left(1 - \frac{\overline{K}^W}{\overline{K}^P}\right) - b\frac{\overline{H}^W}{F}\left(1 - \frac{\overline{H}^P \overline{K}^W}{\overline{H}^W \overline{K}^P}\right)}, \quad (8)$$

in which

$$A = \frac{\overline{H}^P \overline{K}^W [1 + b(1-\Theta)\tilde{\delta}]}{\overline{K}^P \overline{H}^W (1 - b\Theta\tilde{\delta})}.$$

The optimal coordinate ℓ_0 is determined by successive approximation on solving (8) with (3) as a function of C_F, Θ, $\overline{H}^P$, $\overline{K}^P$, $\overline{H}^W$, $\overline{K}^W$, L.

If we assume that

$$\tilde{\delta} \ll 1,\ \overline{H}^P = \overline{H}^W = H,\ \overline{K}^W = \overline{K}^P = K,\ \frac{H}{F} \ll 1, \quad (9)$$

Eq. (8) becomes a standard relation [6]:

$$y_0 = 1 - \Theta. \quad (10)$$

Eq. (10) has been derived [6] from equality for c_F and c_f for $c_F \ll 1$ and obedience to (9).

We now determine the maximum separating power, where for simplicity we assume that (9) and (10) are obeyed. Then [6]

$$\delta U = \frac{H^2}{4K} L \frac{2[1-\exp(-t)]^2}{t}. \tag{11}$$

Here $t = \Theta(1-\Theta)\chi E$; $\chi = F/H$; $E = HL/K$; $H^2/4K$ is the specific separating power, and the maximum for the relative separating power $\delta U/[(H^2/4K)L]$ is given by

$$2t_0 \exp(-t_0)[1-\exp(-t_0)] = 0,$$

whose solution is

$$t_0 \simeq 1.257 = [\Theta(1-\Theta)\chi E]_0. \tag{12}$$

We substitute (12) into (11) to get the maximum relative separating power for a short column, which does not exceed 81%, which agrees with [7].

One can rewrite (12) as follows on the basis that $\Theta(1-\Theta) \leqq 1/4$:

$$(\chi E)_0 \gtrsim 5. \tag{13}$$

Approximate equality applies for symmetrical operation ($\Theta = 0.5$); (13) implies that in the unsymmetrical state ($\Theta \neq 0.5$), the optimum feed has the lower bound $5H/E_0$, and for a given F_0, there are two unsymmetrical states for which Θ_0 is defined by

$$\Theta_0^2 - \Theta_0 + \frac{1.257}{\chi E_0} = 0. \tag{14}$$

Figure 1 shows the optimum tap-off part length $(\ell/L)_0$ from (8) as a function of Θ for various c_F and χ. Here ℓ_0 must be determined on the basis of the working concentration range. For example, for $\Theta = 0.5$ and $c_F \ll 1$, the optimum feed coordinate lies above the central plane of the column, while if c_F is close to one, it lies below it. As χ increases, the dependence of the optimum length on the concentration weakens.

Figure 2 gives the enrichment as a function of the feed coordinate for various E in a thermal-diffusion column, whose working parameters were taken from [3]. This agrees well with the analogous result from [3], i.e., $\delta U = \delta U_0$ for $c_F = c_f$ with the [3] working conditions.

In general, (10) is not obeyed, and $c_F = c_f$ does not correspond to maximum separating power. In other words, if δU is maximal, there may be local mixing at the feed coordinate,

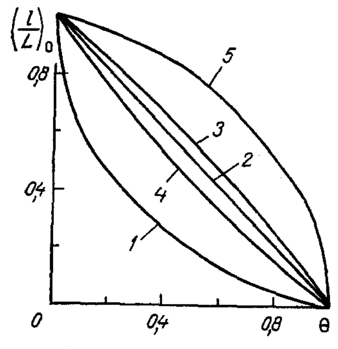

Fig. 1

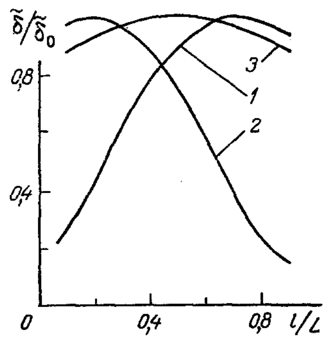

Fig. 2

Fig. 1. Optimum feed coordinate $(\ell/L)_0$ is a function of flow separation factor Θ for various c_F and $\chi = F/H$: 1) $c_F = 1.0 \cdot 10^{-3}$, $\chi = 2$; 2) $c_F = 5.0 \cdot 10^{-1}$, χ any value; 3) $c_F = 9.99 \cdot 10^{-1}$, $\chi = 2$; 4) $c_F = 1.0 \cdot 10^{-3}$, $\chi = 10$; 5) $c_F = 9.99 \cdot 10^{-1}$, $\chi = 10$ for $E = 5.88$.

Fig. 2. Relative enrichment $\tilde{\delta}/\tilde{\delta}_0$ as a function of feed coordinate in a thermal-diffusion column ($\chi = 2$): 1) $c_F = 1.0 \cdot 10^{-3}$, $\Theta = 5.0 \cdot 10^{-2}$, $E = 5.88$; 2) $c_F = 9.99 \cdot 10^{-1}$; $\Theta = 9.5 \cdot 10^{-1}$, $E = 5.88$; 3) $c_f = 5.0 \cdot 10^{-1}$, $\Theta = 0.5$; $E = 1$.

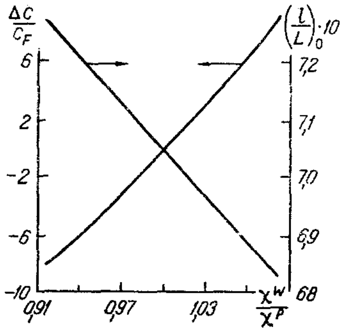

Fig. 3. Optimal feed coordinate (ℓ/L) and relative mixing $(c_F - c_f)/c_F$ in the feed section as functions of χ^W/χ^P: $c_F = 1.0 \cdot 10^{-3}$; $\Theta = 4.8 \cdot 10^{-2}$; $F/\overline{H}^W = 2$; $H^W L/K^W = 5.88$. $\Delta c/c_F$, %.

but here the combination of $\overline{H}^W$, $\overline{K}^W$ and $\overline{H}^P$, $\overline{K}^P$ in $\tilde{\delta}$ from (3) is such that the sum of the separating powers for the tap-off and discard parts will be maximal.

Figure 3 shows ℓ_0 and the relative mixing $\Delta c/c_F = (c_F - c_f)/c_F$ at the feed section as functions of χ^W/χ^P, which characterizes the difference in conditions in the discard and tap-off parts. The optimum feed coordinate from (8) and the corresponding maximum δU can result in local mixing being appreciable. It is then incorrect to use the method of calculating ℓ_0 given in [8], which is based on the condition $c_F = c_f$.

NOTATION

$P = \Theta F$, c^+; $W = (1 - \Theta)$; F, c^-; F, c_F, target-component fluxes and concentrations in the sampling, discard, and feed sections; Θ, flow division factor; δU, separating power; $V(c)$, separation potential; c, concentration averaged over the cross section; c_f, concentration within the apparatus in the feed plane $(z = L - \ell)$; z, longitudinal coordinate; $y = \ell/L$; L, column length; ℓ, tap-off part length; $\overline{H}^W$, $\overline{K}^W$, $\overline{H}^P$, $\overline{K}^P$, coefficients in (1) averaged correspondingly over the heights of the discard and tap-off parts; $\chi = F/H$; $E = HL/K$; $a = c_F^2$, $b = 1 - 2c_F$, linearization constants; $\tilde{\delta}$, enrichment function. Superscripts: 0, maximum separating power; P, tap-off part; W, waste part.

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